Math 3242 - Matrix Arithmetic Worksheet

DEF

1. A **matrix** is a rectangular array of numbers, called **entries**, organized into **rows** and **columns**.
2. The **size** of a matrix with m rows and n columns is denoted by $m \times n$ (rows always precede columns).
3. A matrix with only one column ($m \times 1$) is called a **column vector**.
4. A matrix with only one row ($1 \times n$) is called a **row vector**.
5. A matrix with the same number of rows as columns ($n \times n$) is called a **square matrix**.

We will denote matrices by upper case letters (A , B , C) and vectors by lower case letters with arrows over them ($\vec{a}$, $\vec{b}$, $\vec{c}$).

Example 1 Determine the sizes of the matrices.

$$\begin{bmatrix} 4 & 3 \\ 0 & 2 \\ -6 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & -1 & 6 \end{bmatrix} \qquad \begin{bmatrix} 4 & 0 & 2 & 3 \\ -3 & 15 & 1 & 8 \end{bmatrix} \qquad \begin{bmatrix} 2 & 8 & 1 \\ 12 & 23 & 4 \\ 5 & 5 & -7 \end{bmatrix} \qquad \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$

Matrix Arithmetic

We will often need to refer to specific entries in a matrix, so we introduce the following notation: the entry in the i^{th} row and j^{th} column of the matrix A is denoted by a_{ij}. The lower case letter used to denote the entries of a matrix always match the upper case letter used to denote the matrix itself. If we want to be perfectly clear, we write $A = [a_{ij}]$ meaning " A is a matrix with entries a_{ij} ".

Matrix Addition/Subtraction

If A and B are matrices of the *same size (*matrices of different sizes cannot be added or subtracted), then $A \pm B$ is the matrix created by adding/subtracting the corresponding entries from each matrix. In other words,

$$A + B = [a_{ij} + b_{ij}] \qquad \text{and} \qquad A - B = [a_{ij} - b_{ij}]$$

Scalar Multiplication

If A is a matrix and c is any constant, then cA is the matrix created by multiplying every entry of A by c. In other words,

$$cA = [ca_{ij}]$$

Example 2 For the following matrices perform the indicated operation, if possible.

$$A = \begin{bmatrix} 4 & 0 & 2 & 3 \\ -3 & 15 & 1 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 3 & 2 & 7 \\ 2 & 1 & -1 & -23 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 7 & 3 \\ -4 & 0 & 5 \\ 3 & 6 & 12 \end{bmatrix}$$

a). $A + B$

d). $3C$

b). $B - A$

e). $3A - 2B$

c). $C + A$

f). $-B + 2C$

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Dot Product

If $\vec{a}$ and $\vec{b}$ are vectors of length n, then the **dot product** $\vec{a} \cdot \vec{b}$ is the *number* given by $\sum_{i=1}^{n} a_i b_i$.

Matrix Multiplication

If A is a $m \times p$ matrix and B is a $p \times n$ matrix then the **matrix product** AB is a $m \times n$ matrix whose ij^{th} entry is the dot product of the i^{th} row of A with the j^{th} column of B.

Matrix Transpose

If A is a $m \times n$ matrix, then its **transpose** A^T is the $n \times m$ matrix formed by interchanging the rows and columns of A. In other words, the ij^{th} entry of A is the ji^{th} entry of A^T.

Example 3 For the following matrices perform the indicated operation, if possible.

$$A = \begin{bmatrix} 1 & -3 & 0 & 4 \\ -2 & 5 & -8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 5 & 3 \\ -3 & 10 & 2 \\ 2 & 0 & 4 \\ -1 & -7 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 2 \\ 0 \\ -3 \\ 1 \end{bmatrix}$$

a). AB

b). BA

c). CA

d). AC

e). $B^T C$

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Properties of Matrix Arithmetic

DEF 1. The $n \times n$ matrix with one's on its diagonal and zeros everywhere else

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & & \vdots \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}$$

is called the $n \times n$ **identity matrix**. You can think of this as the matrix equivalent of the number 1.

2. The $n \times n$ matrix consisting entirely of zeros

$$0_n = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

is called the $n \times n$ **zero matrix**. This is obviously the matrix equivalent of the number 0.

In both cases we often drop the subscript n when the size of the matrix is either understood or not important.

Properties of Matrix Addition

1. $A + 0 = 0 + A = A$ • Additive Identity
2. $A - A = 0$ • Additive Inverse
3. $A + B = B + A$ • Additive Commutativity
4. $A + (B + C) = (A + B) + C$ • Additive Associativity

Properties of Matrix Multiplication

5. $AI = IA = A$ • Multiplicative Identity
6. $A0 = 0A = 0$ • Multiplicative Inverse
7. $AB \neq BA$ • **Matrix Multiplication is not Commutative**
8. $A(BC) = (AB)C$ • Multiplicative Associativity

Distributive Properties

9. $c(A + B) = cA + cB$ • Scalar Distributivity
10. $(c + d)A = cA + dA$ • Scalar Distributivity
11. $A(B + C) = AB + AC$ • Left Distributivity
12. $(B + C)A = BA + CA$ • Right Distributivity