

Instructions: Write each solution in claim-proof form, even if the solution is short. Make sure your handwriting is legible and that your proofs **use complete sentences**. Provide enough detail so that it is clear to me that you understand why each step of your proof is correct. I will not accept late assignments, so it is in your best interests to submit your homework on time *even if it is incomplete*.

1. (8 points) Solve the linear congruence: $140x \equiv 133 \pmod{301}$.
2. (8 points) Solve the following system of linear congruences:

$$2x \equiv 1 \pmod{5}$$

$$3x \equiv 9 \pmod{6}$$

$$4x \equiv 1 \pmod{7}$$

$$5x \equiv 9 \pmod{11}$$

3. (8 points) Use Fermat's Theorem to prove that if $\gcd(a, 133) = \gcd(b, 133) = 1$, then $133 \mid a^{18} - b^{18}$.
4. (8 points) Derive the congruence: $a^{21} \equiv a \pmod{15}$ for all $a \in \mathbb{Z}$.
Hint: You may not be able to use Fermat's Theorem (why not?) but what about its corollary?
5. (8 points) Determine whether 17 is prime by determining whether $16! \equiv -1 \pmod{17}$.