MATH 3221 **HW 3** **Due Tuesday, June 30**

Instructions: Write each solution in claim-proof form, even if the solution is short. Make sure your handwriting is legible and that your proofs **use complete sentences**. Provide enough detail so that it is clear to me that you understand why each step of your proof is correct. I will not accept late assignments, so it is in your best interests to submit your homework on time *even if it is incomplete*.

1. (5 points) Prove that the only prime of the form $n^3 - 1$ is 7. **Hint:** First find a factorization of $n^3 - 1$.
2. (5 points) Prove that the only prime p for which $3p + 1$ is a perfect square is $p = 5$.
3. (5 points) Prove that each integer $n > 11$ can be written as the sum of two composite numbers. **Hint:** If n is even, so that $n = 2k$, then consider $n - 6 = 2(k - 3)$; for odd n, consider $n - 9$.
4. (5 points) Prove that if $a \equiv b \pmod{n}$ and $m \mid n$, then $a \equiv b \pmod{m}$.
5. (5 points) Give a counterexample to the following statement: $a^3 \equiv b^3 \pmod{n} \Rightarrow a \equiv b \pmod{n}$.
6. (5 points) Use congruences to find the remainder when 41^{65} is divided by 7.
7. (5 points) Prove that $111^{333} + 333^{111}$ is divisible by 7.
8. (5 points) For $n \geq 1$, use congruences to show that $43 \mid 6^{n+2} + 7^{2n+1}$.