

Instructions: Write each solution in claim-proof form, even if the solution is short. Make sure your handwriting is legible and that your proofs **use complete sentences**. Provide enough detail so that it is clear to me that you understand why each step of your proof is correct. I will not accept late assignments, so it is in your best interests to submit your homework on time *even if it is incomplete*.

1. (5 points) Show that any integer of the form $6k + 5$ is also of the form $3j + 2$, but not conversely.
2. For $n \geq 1$, establish the following:
 - (a) (5 points) The integer $n(7n^2 + 5)$ is of the form $6k$.
 - (b) (5 points) $5 \mid (3^{3n+1} + 2^{n+1})$. **Hint:** use induction.
3. (5 points) Prove that the product of four consecutive integers is 1 less than a perfect square.
Hint: Recall that $k^2 - 1 = (k - 1)(k + 1)$.
4. Prove the following properties of the greatest common divisor:
 - (a) (5 points) If $\gcd(a, b) = 1$ and $c \mid a$, then $\gcd(b, c) = 1$.
 - (b) (5 points) If $\gcd(a, b) = 1$, then $\gcd(a^2, b^2) = 1$. **Hint:** First show that $\gcd(a, b^2) = 1$.
5. Use the Euclidean Algorithm to find $\gcd(a, b)$. Then find integers x and y satisfying $\gcd(a, b) = ax + by$.
 - (a) (5 points) $a = 119, b = 272$.
 - (b) (5 points) $a = 1769, b = 2378$.