

The following is a famous fallacy involving mathematical induction. Often referred to as the Horse Paradox, this convincing (but critically flawed) argument is an attempt to prove the statement ‘all horses are the same color’. Can you identify what is wrong with the argument? (5 points)

Claim: For all $n \geq 1$, in any group of n horses, the horses must be the same color.

Proof: For $n = 1$, the group only consists of a single horse, so obviously all horses in the group are the same color.

Now suppose that the claim is true for some arbitrary n ; that is, we suppose that in any group of n horses, the horses are all the same color. We want to show that this implies that the claim holds for $n + 1$.

Consider any group of $n + 1$ horses. Temporarily remove a horse, we’ll call it horse A, from the group so that you are left with a group containing n horses. By our supposition, these horses are all the same color. Now replace horse A and remove a different horse, horse B. The remaining group contains n horses, so again they must all be the same color. But we’ve already shown that horse B is also of this color, so we can conclude that all $n + 1$ horses are the same color.

Thus the claim is true for all $n \geq 1$ by induction.