Math 1730 — Exam 1 Name _____

Instructions

Each question is worth 5 points. Mark all answers on the provided answer sheet.

You may not use calculators.

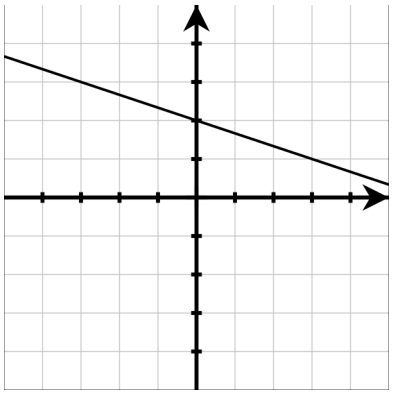
All answers are given in exact form. For example,

$2+\sqrt{5}$ not 4.236.

If a question is unclear, ask me.

Find the following.

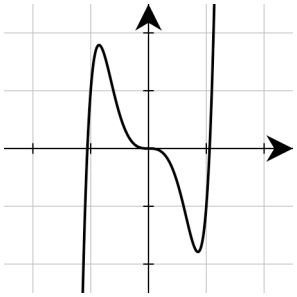
1. Find the linear function whose graph is given below. (Hint: Think about how you would graph each function and see if it differs from the picture)



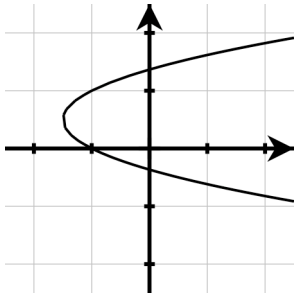
A. $f(x)=-\frac{1}{3}x+2$

2. Which of the following are graphs of functions?

Graph 1



Graph 2



A. Graph 1 is a function

Graph 2 fails the vertical line test.

3. Find the domain of $f(x)=\dfrac{x-3}{x^3-5x^2+6x}$

C. $\mathbb{R}-\{0,2,3\}$

Factoring the denominator, $f(x)=\dfrac{x-3}{x(x-3)(x-2)}$.

Thus we need $x(x-3)(x-2)\neq 0$, so $x\neq 0,2,3$.

Therefore the domain of $f(x)$ is $\mathbb{R}-\{0,2,3\}$.

4. Find the slope of the line containing the points $(3,-1)$ and $(-2,2)$.

C. $-\frac{3}{5}$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{(2)-(-1)}{(-2)-(3)}=-\frac{3}{5}$$

5. Find an equation of the line passing through $(3,-4)$ that is perpendicular to the line $2y+5=x$.

A. $y=-2x+2$

Rewriting $2y+5=x$ in slope-intercept form, we see that $y=\frac{1}{2}x-\frac{5}{2}$ has slope $m_1=\frac{1}{2}$. Then any perpendicular line has slope $m=-\dfrac{1}{m_1}=-2$. Using the point-slope formula we get the following equation:

$$\begin{aligned} y-(-4) &= -2\big(x-(3)\big) \\ y+4 &= -2x+6 \\ y &= -2x+2 \end{aligned}$$

6. For $f(x)=2x^2-3$ and $g(x)=\sqrt{x-1}$, find $(f\circ g)(x)$.

D. $(f\circ g)(x)=2x-5$

$$\begin{aligned} (f\circ g)(x) &= f\big(g(x)\big) \\ &= f\left(\sqrt{x-1}\right) \\ &= 2\left(\sqrt{x-1}\right)^2-3 \\ &= 2(x-1)-3 \\ &= 2x-5 \end{aligned}$$

7. Write an equation for a function whose graph has the same shape as $f(x)=\sqrt{x+2}$, but is shifted up 3 units and left 2 units.

B. $g(x)=\sqrt{x+4}+3$

$$\begin{aligned} g(x) &= f(x+2)+3 \\ &= \left(\sqrt{(x+2)+2}\right)+3 \\ &= \sqrt{x+4}+3 \end{aligned}$$

8. Find the zero(s) of $f(x)=x^2+x-1$

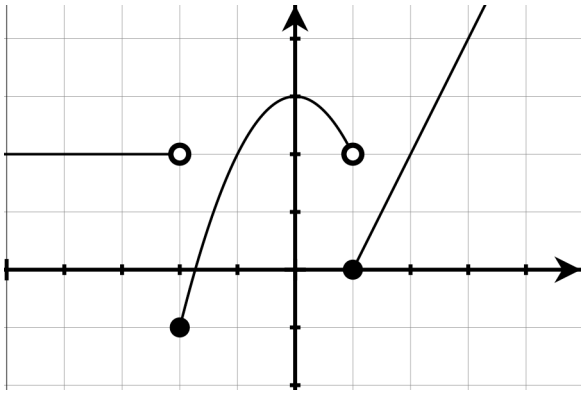
B. $\dfrac{-1\pm\sqrt{5}}{2}$

$$x^2+x-1=0$$

$$x=\frac{-1\pm\sqrt{1-4(1)(-1)}}{2(1)}$$

$$x=\frac{-1\pm\sqrt{5}}{2}$$

Use the following graph to answer the next two questions.



9. On what intervals is the function graphed above increasing?

C. $(-2,0)$ and $(1,\infty)$

The function is clearly constant on $(-\infty,-2)$, increasing on $(-2,0)$, decreasing on $(0,1)$, and increasing again on $(1,\infty)$.

10. Determine which piecewise function has the graph shown above. (Hint: Think about how you would graph each function and see if it differs from the picture)

B. $$f(x)=\begin{cases} 2, & x<-2 \\ -x^2+3, & -2\le x<1 \\ 2x-2, & x\ge 1 \end{cases}$$

From the graph it is clear that $f(-2)=-1$. This rules out answer choices A and C immediately. Also from the graph, $f(-1)=2$, so we can also rule out D.

11. Solve $x^2+4x=32$.

A. $x=-8,4$

$$x^2+4x-32=0$$
$$(x+8)(x-4)=0$$
$$x=-8,4$$

12. Which of the following is an intermediate step when completing the square on $2x^2-8x+7=0$?

B. $(x-2)^2=\frac{1}{2}$

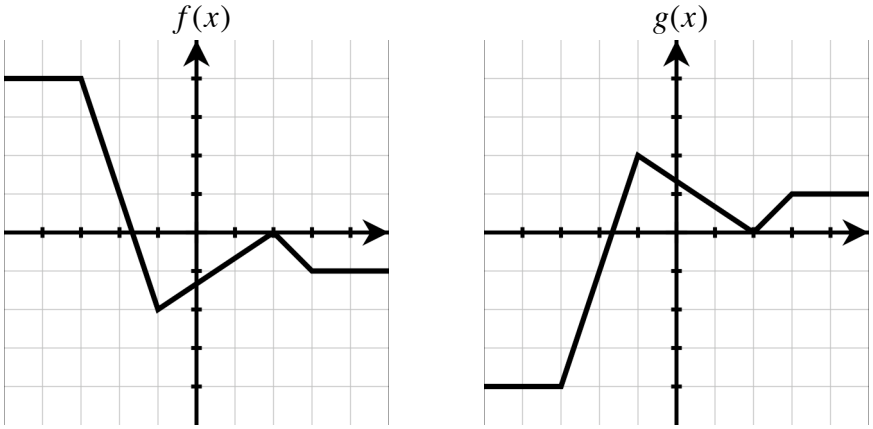
$$2(x^2-4x)+7=0$$
$$2(x^2-4x+4)+7-2(4)=0$$
$$2(x-2)^2-1=0$$
$$(x-2)^2=\frac{1}{2}$$

13. Solve $x^6 + 26x^3 - 27 = 0$. Hint: it is reducible to quadratic.

C. $x = -3, 1$

$$\left(x^3\right)^2 + 26\left(x^3\right) - 27 = 0 \quad \rightarrow \quad u = x^3$$
$$u^2 + 26u - 27 = 0$$
$$(u + 27)(u - 1) = 0$$
$$u = -27, \quad u = 1$$
$$x^3 = -27, \quad x^3 = 1$$
$$x = -3, \quad x = 1$$

14. The graph of a function $f(x)$ is shown on the left below. The graph of a function $g(x)$ is shown on the right. Given what you know about transformations, how can we express $g(x)$ in terms of $f(x)$?



A. $g(x) = -f(x)$

The graph of $g(x)$ is just the reflection of $f(x)$ about the x-axis, thus $g(x) = -f(x)$.

Simplify (answers should be of the form $a + bi$ where a and b are real numbers).

15. $\sqrt{-300}$

C. $10\sqrt{3}i$

$$\sqrt{-300} = i\sqrt{300}$$
$$= 10\sqrt{3}i$$

16. $\dfrac{3+2i}{5-i}$

A. $\frac{1}{2}+\frac{1}{2}i$

$$\frac{3+2i}{5-i}=\frac{3+2i}{5-i}\cdot\frac{5+i}{5+i}$$
$$=\frac{15+3i+10i+2i^2}{25-5i+5i-i^2}$$
$$=\frac{13+13i}{26}$$
$$=\frac{1}{2}+\frac{1}{2}i$$

17. i^{103}

C. $-i$

$$i^{103}=i^3=-i$$

Find the vertex of the graph of the function.

18. $f(x)=3x^2-12x+16$

D. $(2,4)$

$$f(x)=3(x^2-4x)+16$$
$$=3(x^2-4x+4)+16-3(4)$$
$$=3(x-2)^2+4$$
vertex is $(2,4)$

Carry out the following polynomial divisions using the method specified.

19. $\dfrac{3x^4-2x^2+3x-6}{x^3+x^2+2x-1}$ using long division.

C. $3x-3+\dfrac{-5x^2+12x-9}{x^3+x^2+2x-1}$

$$\begin{array}{r} 3x\ -\ 3 \\ x^3+x^2+2x-1\ \overline{)\ 3x^4+0x^3-2x^2+3x-6} \\ \underline{3x^4+3x^3+6x^2-3x}\quad\downarrow \\ -3x^3-8x^2+6x-6 \\ \underline{-3x^3-3x^2-6x+3} \\ -5x^2+12x-9 \end{array}$$

20. $\dfrac{3x^3 - 8x^2 - 11x - 4}{x - 4}$ using synthetic division.

A. $3x^2 + 4x + 5 + \dfrac{16}{x - 4}$

$$\begin{array}{r|cccc} 4 & 3 & -8 & -11 & -4 \\ & & 12 & 16 & 20 \\ \hline & 3 & 4 & 5 & 16 \end{array}$$

Extra Credit (10 points)

21. Use synthetic division to find $f(5)$, where $f(x) = -x^4 + 28x^2 - 21x + 34$.

C. $f(5) = 4$

$$\begin{array}{r|ccccc} 5 & -1 & 0 & 28 & -21 & 34 \\ & & -5 & -25 & 15 & -30 \\ \hline & -1 & -5 & 3 & -6 & 4 \end{array}$$

Thus, by the Remainder Theorem, $f(5) = 4$.

22. We expect the graph of $f(3x)$ to be a graph of $f(x)$ which has been:

D. horizontally shrunk

Multiplication of the input by 3 corresponds to horizontal stretching by a factor of one-third (shrinking).