

## 5. Equations Involving Radicals

While we aren't going to define what it means to be a "radical function" (i.e. a function involving radicals) being able to solve equations involving radicals is a necessary skill which is too often forgotten.

**The Principle of Powers:** For any positive integer  $n$ , if  $a = b$  then  $a^n = b^n$ .

Consider the radical equation  $\sqrt{4x+5} + \sqrt{x+5} = 3$ .

**Step 1:** Solve for one radical (if necessary).

$$\sqrt{4x+5} = 3 - \sqrt{x+5}$$

**Step 2:** Using the Principle of Powers, square both sides. This will rid us of the radical we just solved for.

$$\begin{aligned}(\sqrt{4x+5})^2 &= (3 - \sqrt{x+5})^2 \\4x+5 &= 9 - 3\sqrt{x+5} - 3\sqrt{x+5} + (\sqrt{x+5})^2 \\4x+5 &= 9 - 6\sqrt{x+5} + (x+5)\end{aligned}$$

**Step 3:** If necessary, repeat steps 1 and 2 to get rid of another radical in the equation. Repeat this until all radicals are gone.

We have another radical in the equation so we repeat the steps above.

1. Solve for the radical

$$\begin{aligned}4x+5 &= 9 - 6\sqrt{x+5} + (x+5) \\3x-9 &= -6\sqrt{x+5}\end{aligned}$$

2. Square both sides.

$$\begin{aligned}(3x-9)^2 &= (-6\sqrt{x+5})^2 \\9x^2 - 54x + 81 &= 36(x+5)\end{aligned}$$

**Step 4:** Solve for  $x$ .

$$\begin{aligned}9x^2 - 90x - 99 &= 0 \\x^2 - 10x - 11 &= 0 \\(x-11)(x+1) &= 0 \\x &= -1, 11\end{aligned}$$

**Step 5:** **Check all solutions.** You cannot assume that all solutions from step 4 are correct.

$$x = -1 \quad \sqrt{4(-1)+5} + \sqrt{(-1)+5} = 1+2 = 3$$

OK

$$x = 11 \quad \sqrt{4(11)+5} + \sqrt{(11)+5} = 7+4 = 11$$

NOT A SOLUTION

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**Note:** Similar to rational equations, the method for solving radical equations often requires the checking of solutions. In particular, we must check solutions whenever we use the Principal of Powers to raise both sides of an equation to even powers. By raising unknown quantities to even powers, we are potentially “adding” solutions which may not exist. Very simply, this is because raising things to even powers means you lose information about what was negative and what wasn’t.

**Examples on the board:**

1. Solve  $\sqrt[5]{2x-3} - 1 = 1$

2. Solve  $\sqrt{6x+6} = 5 + \sqrt{21-4x}$ .